Business Calculus Test 3 Review Answers

Dr. Graham-Squire, Summer Session 1, 2012

1. Find the absolute maximum and minimum (if they exist) of the function $g(x) = x\sqrt{4-x^2}$ on the interval [0,2].

   Ans: Absolute min of 0 at both $x = 0$ and $x = 4$, abs. max of 1.89 at $x = \sqrt{4/3}$.

2. A rectangular box is to have a square base and a volume of 20 ft^3. If the material for the base costs 30 cents/ft^2, the material for the top costs 20 cents/ft^2, and the material for the sides costs 20 cents/ft^2, determine the dimensions of the box that give a minimum cost. Check your answer to make sure it is a minimum.

   Ans: 2.52 ft by 2.52 ft by 3.15 ft (where the 3.15 is the height).

3. The number of internet users in China is approximated by the function

$$N(t) = 94.5e^{0.2t} \qquad (1 \le t \le 6)$$

   where $N(t)$ is measured in millions and t is years with $t = 1$ being 2005.

   (a) How many users are there in 2010? 313, 751, 049 users.

   (b) When did the number of users equal 190,300,000? When $t = 3.5$, so approximately the middle of 2007.

4. Expand and simplify the expression $\ln \dfrac{x^2 \cdot e^{3x}}{\sqrt{x}(1+x)^2}$.

   Ans: $2\ln x + 3x - \frac{1}{2}\ln x - 2\ln(1+x)$

5. Find the interest rate needed for an investment of \$4000 to double in 5 years if interest is compounded continuously.

   Ans: 13.86%

6. Find $f'(x)$ if $f(x) = \ln \dfrac{e^{3x}+4}{8}$.

   Ans: $f'(x) = \dfrac{3e^{3x}}{e^{3x}+4}$.

7. The percentage of alcohol in a person's bloodstream t hr after drinking 8 fluid oz of whiskey is given by

$$A(t) = 0.23te^{-0.4t}$$

   (a) How fast is the percentage changing after 1 hour? 0.0925

   After 4 hours? -0.0279

   (b) Use calculus to find at what value of t is the percentage at a *maximum*. When $t = 2.5$.

   What is the percentage at that time? 0.21 (Way above the legal limit of 0.08).

8. Use logarithmic differentiation to find $f'(x)$ if $f(x) = x^{2x}$.

   Ans: $f'(x) = x^{2x}(2\ln x + 2)$.

9. The element Grahamsquireium has a half-life of 250 years. Given a 100 gram sample, how much of it will be left after 300 years?

   Ans: 43.53 grams.

10. Find the indefinite integral $\displaystyle\int x\left(\sqrt{x} + \frac{3}{x^2} - \frac{2e^x}{x}\right) dx$.

    Ans: $\displaystyle\frac{2}{5}x^{5/2} + 3\ln x - 2e^x + C$